

## A Resolution to Cherenkov-like Radiation of OPERA Neutrinos

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### Abstract

The OPERA collabotation has reported evidence of superluminal neutrinos with a mean energy 17.5 GeV ranging up to 50 GeV. However, the superluminal interpretation of the OPERA results has been recently refuted theoretically by Cherenkov-like radiation. We discuss a loophole of this argument from the kinematical viewpoint and find it possible to avoid the Cherenkov-like radiation of the OPERA neutrinos. The key idea of our argument is to admit the fact that the neutrinos travel faster than the observed speed of light while they do slower than the true speed of light in vacuum so strictly speaking they are not superluminal but subluminal. Moreover, we present a model where these two velocities of light can be constructed by taking account of influences from dark matters near the earth.

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# 1 Introduction

The OPERA experiment has recently announced a remarkable discovery that neutrinos from CERN to Gran Sasso travel faster than the speed of light [1]. Soon after this sensational announcement, there have been a lot of activities to attempt to understand this interesting phenomenon of superluminal neutrinos from the theoretical viewpoint. As a consequence of such theoretical studies, it has turned out that there are two theoretical challenges to the OPERA experimental results, which are the bremsstrahlung effects [2] and pion decay [3, 4, 5]. Since the two problems stem from the same cause that the OPERA neutrinos are superluminal, we focus on only the bremsstrahlung effects in this article. We will see that the problem of pion decay is also automatically solved by our theory.

Let us start with an explanation of what we call, the bremsstrahlung effects or Cherenkov-like radiation of superluminal neutrinos. On the way from CERN to Gran Sasso, the very effect of superluminal propagation of neutrinos would have caused some distortions in the beam of neutrinos owing to Cherenkov-like radiation and severely depleted the higher-energy neutrinos, thereby making it impossible to observe neutrinos with more than 12.5 GeV energy. This theoretical result is obviously against the OPERA results where a lot of high-energy neutrinos above 12.5 GeV are observed as seen in the OPERA paper [1] "Data were then split into two bins of nearly equal statistics, including events of energy lower or higher than 20 GeV. The mean energies of the two samples are 13.9 and 42.9 GeV."

More recently, stimulated with the above theoretical observation, ICARUS group has analyzed their data and found that "the neutrino energy distribution of the ICARUS events in IAr agrees with the expectations from the Monte Carlo predictions from an unaffected energy distribution of beam from CERN. Our results therefore refute a superluminal interpretation of the OPERA result according to the Cohen and Glashow prediction for a weak current analog to Cherenkov radiation" [6].

In order to make a physically plausible theory of the OPERA results, one has to overcome this issue at all events. It is our aim in this article to present one solution to this problem. To do so, it is worth keeping in mind that since physics of Cherenkov-like radiation of superluminal neutrinos is purely in the kinematical regime, one has to propose a not dynamical but kinematical solution to this problem which would forbid this process from the kinematical reasons.

In addition to it, we cannot help accepting the fact that according to Cohen and Glashow if a neutrino could travel faster than the speed of light in vacuum as claimed by OPERA, superluminal neutrinos rapidly lose energy via pair bremsstrahlung so few neutrinos with energies in excess of 12 GeV are detected at Gran Sasso.

In this article, let us suppose that the OPERA results are correct at any rate even if further experimental scrutiny is surely needed. Then, a question naturally arises: "Is there a way out to reconcile the OPERA results which insist on the superluminal propagation of neutrinos, with the bremsstrahlung effects which refute a superluminal interpretation of the OPERA results by the kinematics?" Below we wish to present a solution to this question. The key idea is to introduce two kinds of speeds of light in such a way that the OPERA

neutrinos are not *superluminal* but *subluminal* under the true velocity of light.

This article is organized as follows: In the next section, we will review the work by Cohen and Glashow. In Section 3, we explicitly make a model which has two speeds of light because of interaction between the gauge field and a Galileon-type of scalar field. Section 4 is devoted to discussion.

## 2 Review of bremsstrahlung effects and its resolution

Let us begin with a review of the bremsstrahlung effects of superluminal neutrinos [2]. This study is based on an old analysis of Ref. [7] where many effects of violation of Lorentz invariance have been explored in the framework of Standard Model.

The basic idea behind the work by Cohen and Glashow is that a neutrino traveling faster than the speed of light loses energy by emitting something owing to weak interaction although neutrinos do not carry electric charges (So this process is dubbed Cherenkov-*like*-radiation as well). The most dominant decay process of a muon neutrino, which mainly constitutes the OPERA beam, is found to be  $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$  whose threshold energy reads

$$E = \frac{2m_e}{\sqrt{v_\nu^2 - 1}} \approx 140 \text{ MeV}, \quad (1)$$

for the OPERA neutrinos.

In the high energy limit where mass of an electron and neutrino can be neglected, the rate of bremsstrahlung pair emission  $\Gamma$  and its associated rate of losing energy  $\frac{dE}{dx}$  are calculated to be

$$\begin{aligned} \Gamma &= k' \frac{G_F^2}{192\pi^3} E^5 (v_\nu^2 - 1)^3, \\ \frac{dE}{dx} &= -k \frac{G_F^2}{192\pi^3} E^6 (v_\nu^2 - 1)^3, \end{aligned} \quad (2)$$

where  $k = \frac{25}{448}$ ,  $k' = \frac{1}{14}$  are numerical constants. Then, the mean fractional loss of energy at each step of the bremsstrahlung pair emission is given by

$$\frac{-\frac{dE}{dx}}{\Gamma E} = \frac{k}{k'} \approx 0.78, \quad (3)$$

which implies that about  $\frac{3}{4}$  of the neutrino energy is reduced at one step of emission.

Moreover, by integrating  $\frac{dE}{dx}$  over a distance  $L$ , the final neutrino energy  $E$  is expressed in terms of its initial one  $E_0$  by

$$\frac{1}{E^5} = \frac{1}{E_0^5} + 5k \frac{G_F^2}{192\pi^3} (v_\nu^2 - 1)^3 L. \quad (4)$$

By applying this formula to the case of the OPERA experiment, the terminal energy of a neutrino at the OPERA detector is about 12.5 GeV. In other words, few neutrinos reach the

detector with energies in excess of 12.5 GeV. Unfortunately, the OPERA detector observes neutrinos with the mean energy of 17.5 GeV ranging up to 50 GeV [1], so the calculation rules out an interpretation that the OPERA neutrinos are superluminal. Incidentally, the ICARUS group has recently reanalyzed their data obtained in 2010 and found that no Cherenkov-like radiation has been detected in ICARUS [6].

Where is there a loophole in the above argument? It is true that we cannot help accepting the conclusion by Cohen and Glashow as long as a neutrino is superluminal. Thus, the only way out is to consider that the OPERA neutrinos are not superluminal but subluminal, but then how can we make them stay subluminal, which appears to be against the OPERA report?

As an implicit assumption of the argument by Cohen and Glashow, the speed of light is considered as a universal constant and does not receive any influences from the environment. However, it has been already pointed out that there is the possibility that the velocity of the photons might be dependent on energy from the HERA Compton polarization data [8].<sup>2</sup> It might be therefore reasonable to conjecture that the speed of light would receive some influences from surroundings and consequently the observed speed of light in vacuum might become smaller than the true speed of light in vacuum.

Put differently, if the group velocity  $v_\nu$  of the OPERA neutrinos is between the observed velocity  $c_0$  of light and the true one  $c$ , i.e.,  $c_0 < v_\nu < c$ , the OPERA neutrinos can be regarded as subluminal since the property of superluminality or subluminality is defined by using the true velocity  $c$  of light in vacuum.

Then, the problem of the bremsstrahlung effects is converted to a different problem: Can we construct a model which has two kinds of velocities of light, which satisfy the relation  $c_0 < c$ , without conflicting with special relativity? In the next section, we shall present such a model based on the recent works [9, 10, 11, 12, 13, 14].

### 3 A model of two velocities of light

We shall begin with the Lagrangian density of our theory :<sup>3</sup>

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2M_*^4}\partial^\nu\pi\partial^\alpha\pi F_{\mu\nu}F^\mu{}_\alpha, \quad (5)$$

where  $M_*$  is a mass scale which controls the strength of the coupling between the Galileon-type of scalar field  $\pi$  [15] and the abelian gauge field  $A_\mu$ . The gauge field strength  $F_{\mu\nu}$  is defined by  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  as usual. In the absence of the  $\pi$  field, the photon travels at the speed of light in vacuum since the gauge field satisfies the conventional Maxwell equations. In the theory at hand, it is essential to define this velocity, which we denote as  $c$ , as the *true* speed of light in vacuum, which appears in various formulae in special relativity. Thus, the causal structure should be determined on the basis of this velocity  $c$  and the flat metric  $\eta_{\mu\nu}$ .

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<sup>2</sup>We identify the velocity of the photons with that of light.

<sup>3</sup>We make use of the flat metric  $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$  for raising or lowering indices.

Now it is easy to rewrite (5) as

$$\mathcal{L} = -\frac{1}{4}(\eta^{\mu\alpha} - \frac{1}{M_*^4}\partial^\mu\pi\partial^\alpha\pi)(\eta^{\nu\beta} - \frac{1}{M_*^4}\partial^\nu\pi\partial^\beta\pi)F_{\mu\nu}F_{\alpha\beta}, \quad (6)$$

thereby making it possible to read out an effective metric

$$g_{(A)}^{\mu\nu} = \eta^{\mu\nu} - \frac{1}{M_*^4}\partial^\mu\pi\partial^\nu\pi, \quad (7)$$

along which the photon propagates.

Next, let us consider the spherical symmetric configuration for the scalar

$$\pi = \frac{\alpha}{r}, \quad (8)$$

where  $\alpha$  is a constant [11]. Note that this configuration breaks the Lorentz invariance spontaneously. Then, it turns out that the effective space-time on which the photon propagates has the line element

$$ds^2 \equiv g_{(A)\mu\nu}dx^\mu dx^\nu = -dt^2 + \frac{1}{1 - \frac{\alpha^2}{M_*^4}\frac{1}{r^4}}dr^2 + r^2 d\Omega_2^2, \quad (9)$$

where  $d\Omega_2^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$ .

It is then straightforward to derive an effective velocity  $c(r)$  of the photon for the fixed angles  $(\theta, \phi)$

$$c(r) \equiv \frac{dr}{dt} = \left(1 - \frac{\alpha^2}{M_*^4}\frac{1}{r^4}\right)^{\frac{1}{2}} c, \quad (10)$$

where for convenience the *true* velocity  $c$  of light in vacuum is recovered. Note that at the spatial infinity  $r \rightarrow \infty$ , the effective velocity coincides with the *true* velocity  $c$ , that is,

$$\lim_{r \rightarrow \infty} c(r) = c. \quad (11)$$

Furthermore, note that the difference between  $c$  and  $c(r)$  is positive-definite as long as the constant  $\alpha$  is non-zero

$$c - c(r) = \frac{\alpha^2}{2M_*^4}\frac{1}{r^4} > 0, \quad (12)$$

which holds when  $\frac{\alpha^2}{M_*^4}\frac{1}{r^4} \ll 1$ .

On our earth, the observed velocity of light is given by  $c(r)$  when the photons are located at the place whose distance in the radial direction is  $r$  from the center of the earth. If we assume that the speed of a neutrino, denoted as  $v_\nu$ , takes a definite value and is smaller than  $c$  but larger than  $c(r)$

$$c(r) < v_\nu < c, \quad (13)$$

we would have a superluminal neutrino for the velocity  $c(r)$  as observed in the OPERA experiment

$$\beta(r) \equiv \frac{v_\nu - c(r)}{c(r)} > 0, \quad (14)$$

whereas we have a subluminal neutrino for the true velocity  $c$

$$\beta \equiv \frac{v_\nu - c}{c} < 0. \quad (15)$$

Recalling that the property of superluminality or subluminality of neutrinos is now defined by using the *true* velocity  $c$  of light in vacuum, the OPERA neutrinos are actually not superluminal but subluminal! Hence, we do not have Cherenkov-like radiation for the OPERA neutrinos at all since we can always take the rest frame for the subluminal neutrino. This is our resolution to the problem of the bremsstrahlung effects of the OPERA neutrinos. It is worthwhile to notice that our solution is purely kinematical as desired.<sup>4</sup>

Let us show that the results of OPERA and SN1987A give us information on the mass scale  $M_*$ . First, let us note that the OPERA result yields a condition for the dimensionless quantity  $\beta(r)$

$$\beta(R_\oplus) \equiv \frac{v_\nu - c(R_\oplus)}{c(R_\oplus)} \approx 2.5 \times 10^{-5}, \quad (16)$$

where  $R_\oplus$  is the radius of the earth and takes the value  $R_\oplus = 6.4 \times 10^8 cm$ .

Next, let us utilize the fact that neutrinos from SN1987A to the earth travels at almost the same velocity as the true velocity of light in vacuum, so we have a relation

$$v_\nu \approx c. \quad (17)$$

With the help of this relation (17), Eq. (16) can be cast to the form

$$\beta(R_\oplus) \approx \frac{\alpha^2}{2M_*^4} \frac{1}{R_\oplus^4}. \quad (18)$$

At this stage, we may assume the scalar field  $\pi$  is sourced by the trace part of the energy-momentum tensor on the earth which does not include the contribution from the scalar field  $\pi$  itself. With this assumption, the numerical constant takes the form in the static case [11]

$$\alpha = \frac{M_\oplus}{M_*} \equiv M_\oplus L_*, \quad (19)$$

where  $M_\oplus$  is the mass of the earth and we have defined the length scale by  $L_* = \frac{1}{M_*}$ .

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<sup>4</sup>Recently, a similar resolution has been also proposed by Nakanishi in Ref. [16]. It is a pity that his interesting article is written only in Japanese.

Using Eq's. (16), (18) and (19),  $L_*$  is calculated as

$$L_* \approx \left( 2 \times 10^{-4} L_{Pl}^4 \frac{R_{\oplus}^4}{R_{\oplus SS}^2} \right)^{\frac{1}{6}} \approx 3 \times 10^{-17} cm, \quad (20)$$

where  $L_{Pl}, R_{\oplus SS}$  are respectively the Planck length and the Schwarzschild radius of the earth, and are explicitly given by  $L_{Pl} = 1.6 \times 10^{-33} cm, R_{\oplus SS} = 0.89 cm$ . Then, it is of interest to notice that  $M_*$  is approximately equivalent to the energy scale where new physics beyond Standard Model appears

$$M_* \approx 1 TeV, \quad (21)$$

above which the coupling between the gauge field and the scalar field becomes strong.

In this context, it is tempting to identify the  $\pi$  scalar field with a candidate of dark matters. Actually, this identification seems to be consistent with the results of OPERA and SN1987A at the same time by the following reasoning: In general, dark matters are expected to have a tendency of localizing near massive objects such as stars and the earth owing to gravitational interaction, compared to empty regions of outer space. The reason why neutrinos from SN1987A to the earth traveled at almost the same velocity as the true velocity of light in vacuum is that since there are not so much dark matters in outer space, the neutrinos from SN1987A propagated at the true velocity  $c$  without interacting with dark matters. On the other hand, since it is expected that there are sufficient dark matters on the earth, the interaction between the photons and dark matters reduces the speed of light on the earth to the smaller observed velocity  $c_0$  from the larger true velocity  $c$ .

## 4 Discussion

In this article, we have presented a resolution to one serious theoretical problem, that is, the bremsstrahlung effects of superluminal neutrinos. Our resolution is on the kinematical grounds and thus strictly forbids the OPERA neutrinos to emit a pair of electron and positron via the bremsstrahlung effects. In this article, we have introduced the Galileon-type of scalar field to interact with the gauge field, but it would be possible to consider the other fields such as spin 2 new tensor field and spin 1/2 spinor field instead of the scalar field.

It is then natural to ask ourselves if our resolution also provides a resolution to the other problems associated with the OPERA results. In particular, it is now known that the other challenging theoretical issue lies in pion decay process [3, 4, 5] where it was found that the decay of charged pion  $\pi^+ \rightarrow \mu^+ + \nu_\mu$ , which is nothing but the neutrino production process in the OPERA experiment, becomes kinematically forbidden for  $E_\nu > 5 GeV$ , which is obviously inconsistent with the OPERA results. It is worth stressing that our resolution also provides a solution to this problem since this decay process is automatically prohibited whenever the OPERA neutrinos are subluminal.

Finally, let us comment on the connection with related works. Moffat has presented a similar idea by taking account of bimetric relativity [17]. Li and Nanopoulos have recently advocated an idea that all the Standard Model particles might be subluminal due to background effects on the basis of a string theory-inspired model [18]. Even if our present theory is very different from these works, it might be interesting to investigate the relation more closely in future.

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